Ice cores preserve valuable information about past environment and climate. To interpret the information within them, it is pivotal to date the ice core. The problem of dating ice cores has been studied for many years, and various methods have been developed. The key methods include (1) layer counting using the seasonality in signals, (2) statistical modeling describing processes such as snow accumulation and plastic deformation of ice, (3) comparison with other dated records, and (4) a combination of these. Currently, none of these methods benefit from statistical modeling.

We are pioneering the combined use of glaciological and statistical models within a Bayesian framework. By formalising the uncertainty in glaciological relations, the dating uncertainty can be derived. This approach is applied to the dating of Antarctic ice cores and then compared to an independent dating derived from layer counting. For the first time, the effects of uncertainty implied by the dating method are investigated for ice core chronologies. This provides valuable insights for the applied community.

**MODEL SET-UP AND ASSUMPTIONS**

Ice cores are typically cut into slices and their chemical and physical properties analysed. We focus on measurements of the oxygen isotope ratio 18O, which is an temperature proxy used to date ice cores.

A glaciological model

- It is assumed that a function $f$ exists which transforms the measurements $Y = (Y_1, Y_2, \ldots )$ of the temperature proxy 18O at depths $d_1, d_2, \ldots$ to observed log-accumulation rates of snow $\dot{Y} = (\dot{Y}_1, \ldots, \dot{Y}_N)$. In particular,

$$f(Y) = A + \epsilon, \quad \dot{Y}_i = f(Y_i)$$

shall be used here (as in [1]), where $A = (A_1, \ldots, A_N)$ is conventionally fixed and $\epsilon = (\epsilon_1, \ldots, \epsilon_N)$. For each slice $i$ the measurement $Y_i$ prevails from depth $d_i$ down to depth $d_{i-1}$, such that the elapsed time $T_i$ is inversely proportional to the accumulation rate. (Let $D = (d_1 - d_0, \ldots, d_N - d_{N-1})$.)

Combining glaciological and statistical modelling

- To quantify the uncertainty in dating ice cores we assume that the observed log-accumulation rates $f(Y)$ are additively related to true log-accumulation rates $A$.

$$P(Y|A) = \prod_{i=1}^N P(Y_i|A_i).$$

(3)

- The errors are assumed to be independent and normally distributed:

$$f(Y) = A + \epsilon, \quad \epsilon_i \sim N(0, \sigma^2).$$

(2)

- Additionally, uncertainty in the parameters $A$ of the relationship $f$ is incorporated.

**Quantities of Interest**

- The posterior distribution of the estimated times covered by each slice $\dot{A}(Y)$ derived from

$$P(\dot{A}|Y) \propto \pi(\dot{A}) \pi(Y|\dot{A}) \propto \pi(\dot{A}) \prod_{i=1}^N P(f(Y_i|A_i), \dot{Y}_i|A_i, \theta) d\theta.$$

(3)

- More interesting is the aggregated posterior distribution of the number of years covered by an ice core $P(\delta|Y)$, which is calculated from (3) using the central limit theorem.

- The effect of different plausible priors on the posterior distributions $P($\dot{A}$|Y)$ and $P(\delta|Y)$ is incorporated.

**RESULTS**

**Prior Uncertainty in Accumulation and the Error**

In order to make the calculation of $P(\delta|Y)$ analytically tractable and to describe a range of possible priors, we consider the log-accumulation rates $\dot{A}$, we tested different normal distributions as well as an improper uniform distribution. The respective distributions for the accumulation rates $\dot{A}$ are depicted in Figure 1 (a). For the error term, different Gamma and log-normal distributions for $\epsilon$ were analysed to cover a plausible range of prior assumptions (see Figure 1 (b)).

**Posterior Distribution of the Number of Years Covered**

The effects of these particular prior distributions on the posterior distribution of the time covered by the core, given the current measurements, are depicted in Figures 2 and 3. In particular, the coloured lines for the posterior distributions $P(\delta|Y)$ in Figure 2 were calculated by using the prior distributions on the accumulation rates of the same colour in Figure 1 (a) together with the log-normal prior distribution $-\log N(0.2, 0.04)$ (depicted in Figure 1 (b)). Similarly, the different line types of the posterior distributions $P(\delta|Y)$ in Figure 3 were calculated by using the corresponding prior distributions on $\sigma$ in Figure 1 (b) together with the log-normal prior distribution $-\log N(-0.7, 0.2^2)$ on the accumulation rates (depicted in Figure 1 (a)).

**DISCUSSION AND FUTURE RESEARCH**

The current research has examined the effect of candidate prior distributions for uncertain quantities of a glaciological model on the posterior distribution of the number of years covered by an ice core. Initial simplifying assumptions had to be made. Future research will use recent snow accumulation data from Automatic Weather Stations as well as recent 18O measurements of snow to

- revise assumption (1) of independent accumulation rates and assumption (2) on the structure of the error and develop more realistic assumptions, and

- substitute each candidate prior distribution by an informative prior distribution.

In order to date ice cores of increased depths, the deformation of ice under the strain of ice which accumulated above it has to be taken into account. Future research will therefore modify the relationship between accumulation and time to $T = \dot{A}D$ in order to take ice sheet thinning into account. Glaciological models can be supported by other dated records. Uncertainty in the posterior distribution of the number of years covered can thus be reduced by including extra prior knowledge on e.g. volcanic eruptions which leave traces in the ice core.

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References: