Hidden Markov Random Field and FRAME Modelling for TCA Image Analysis

Katy Stresco and Francesco Lagona

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Introduction

- Introduction to TCA Image Analysis
- The Model - Hidden Markov Random Field (HMRF)
  - Observable Random Field
  - Markov Random Field (MRF - FRAME)
- Parameter Estimation and Segmentation
  - EM algorithm and MFA
- Results
- Conclusions
Introduction to TCA Image Analysis
TCA Method and Images

- Introduction to TCA Image Analysis
- TCA Method and Images
- TCA image

The Model - HMRF

Subsummary

Parameter Estimation and Segmentation

Results

Conclusions

Thank you!

\[ \text{Hoppa and Vaupel, 2002} \]
TCA Method and Images

- age estimation method

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\[\text{[Hoppa and Vaupel, 2002]}\]
- age estimation method
- inter/intra observer variance and large databases lead to the need of an automatic evaluation
- **TCA** images
  - \(\approx 1016 \times 1300\) pixels
  - gray values \([0, 2^8 - 1]\) or \([0, 2^{12} - 1]\)
  - tooth ring roughly 1-3 \(\mu m\) (5-20 pixel) thick

\[^a\text{[Hoppa and Vaupel, 2002]}\]
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\[ Y : S \rightarrow \mathbb{R}^{N \times M} \]
with \( i \rightarrow Y_i \)

\[ \text{[Li, 2001]} \]
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unknown label image

TCA image $Y$

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\lambda : S \leftrightarrow \mathcal{G}^{N \times M}
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with \( i \) \( \rightarrow \) \( Y_i \) \\
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estimate label image from noise-corrupted observed TCA image

\[ a^{[Li, 2001]} \]
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- Markov Random Field $\Lambda$
- FRAME
- Simulation Example

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- set up a Hidden Markov Random Field (for example$^a$)
- models $\mathcal{Y}$ as mixture

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\[\varepsilon_i = Y_i - \mu \lambda_i\]

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- incorporates the prior knowledge about the image
- describes probability of each pixel $i$ with the help of its neighbors $N(i)$:

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P(\lambda) = \frac{1}{Z} e^{\sum_{i \in S} \phi[(F_T * \lambda)(i)]}
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- filter response \( (F_T * \lambda)(i) \) measures similarity of the neighborhood of each pixel to the filter
- potential function \( \phi \) evaluates the filter responses

\(^a\)[Zhu and Mumford, 1997],[Zhu et al., 1997],[Zhu et al., 1998]
\[
P(\lambda) = \frac{1}{Z} e^{\sum_{i \in S} \phi[(F_T \ast \lambda)(i)]}
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- elegantly combines MRF modelling and filtering theory
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- may be applied to a wide variety of even large scale textures because \( F_T \) accounts for long-range dependencies
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  - use the real part of 2-D Gabor functions

\[ G_{\cos T, \alpha}(x, y) = c \cdot e^{-\frac{(x' \cos \alpha + y' \sin \alpha)^2}{2T^2}} \cos \left( \frac{2\pi x'}{T} \right) \]

\[ x' = x \cos \alpha + y \sin \alpha \]
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  - e.g. \( T = 16, \ \alpha = 0 \)
FRAMES

\[ P(\lambda) = \frac{1}{Z} e^{\sum_{i \in S} \phi[(F_T \star \lambda)(i)]} \]

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Filter Family and Potential Function Specification
- application driven
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\[ G_{\cos T, \alpha}(x, y) = c \cdot e^{-\frac{(rx'^2+y'^2)}{2T^2}} \cos \left( \frac{2\pi}{T} x' \right), \quad x' = x \cos \alpha + y \sin \alpha, \quad y' = -x \sin \alpha + y \cos \alpha \]

- e.g. \( T = 16, \ \alpha = 0 \)
- convolution \( (F_T, \alpha \ast \lambda) \) captures lines of width \( T \) and orientation \( \alpha \)
- choose \( T = \{2, 4, 6, \ldots, 18\} \), fix \( \alpha = 0 \)
\[ P(\lambda) = \frac{1}{Z} e^{\sum_{i \in S} \phi[(F_T \ast \lambda)(i)]} \]

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    \[
    G_{\cos T, \alpha}(x, y) = c \cdot e^{\frac{-(r x' \cos \alpha + y \sin \alpha)^2}{2 T^2}} \cos \left( \frac{2\pi}{T} x' \right), \quad x' = x \cos \alpha + y \sin \alpha, \quad y' = -x \sin \alpha + y \cos \alpha
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  - e.g. \( T = 16, \; \alpha = 0 \)
  - convolution \( (F_T, \alpha \ast \lambda) \) captures lines of width \( T \) and orientation \( \alpha \)
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  - choose simplest cup shaped potential function \( \phi = |.| \)
Simulation Example

\[ P(\lambda) = \frac{1}{Z} e^{\sum_{i \in S} \phi(F_T * \lambda)(i)} \]
Simulation Example

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Gibbs sampler

prior knowledge ('ideal' TCA image)
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REMARK: we don’t want to synthesize perceptual equivalent images but focus on one feature (one filter with one potential function)
Subsummary
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TCA Image

\[ f(Y) = \sum_{\lambda \in G^{N \times M}} P(\lambda) f(Y | \lambda) \]

\[ P(\lambda) = \frac{1}{Z} e^{-\sum_{i \in S} \phi[(F_T \ast \lambda)(i)]} \]

\[ f(Y | \lambda) = \prod_{i \in S} \frac{1}{\sqrt{2\pi\sigma_{\lambda_i}}} e^{-\frac{(Y_i - \mu_{\lambda_i})^2}{2\sigma^{2}_{\lambda_i}}} \]
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TCA Image → HMRF

\[ f(Y) = \sum_{\lambda \in \mathcal{G}^{N \times M}} P(\lambda) f(Y|\lambda) \]

MRF

FRAME

Gaussian

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EM with MFA \[ \hat{\theta}, \hat{T} \]

Label Image
Parameter Estimation and Segmentation
Using EM

- estimate parameters of observable random field: \( \theta = \{ \mu_g, \sigma_g^2 \mid g \in G \} \)
and of MRF: \( T \)
Using EM

- estimate parameters of observable random field: $\theta = \{\mu_g, \sigma^2_g | g \in \mathcal{G}\}$
and of MRF: $T$

- MLE $\{\hat{\theta}, \hat{T}\} = \arg \max_{\{\theta, T\}} L(\theta, T|Y)$

  intractable, because $L(\theta, T|Y) = \sum_{\lambda \in \mathcal{G}^N \times \mathcal{M}} P(\lambda|T)f(Y|\lambda, \theta)$
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- EM\[^{a}\]
  - focusses on complete-data likelihood $L(\theta, T|Y, \lambda)$
  - iterates between
    1.) E-step: $E \left[ \log P(Y, \lambda|\theta, T)|Y, \theta^{(t-1)}, T^{(t-1)} \right]$
    2.) M-step:{\{\theta^{(t)}, T^{(t)}\}} = \arg \max_{\{\theta,T\}} E \left[ \log P(Y, \lambda|\theta, T)|Y, \theta^{(t-1)}, T^{(t-1)} \right]$

  - for Gaussian random field this reduces to three updating formulas

\[^{a}\] [McLachlan and Krishnan, 1997], [Zhang et al., 2001]
Using EM and MFA

\[ \mu_{g}^{(t)} = \frac{\sum_{i \in S} Y_i P \left( \lambda_i = g | Y_i, \lambda_{N(i)}, \theta^{(t-1)}, T^{(t-1)} \right)}{\sum_{i \in S} P \left( \lambda_i = g | Y_i, \lambda_{N(i)}, \theta^{(t-1)}, T^{(t-1)} \right)} \]

\[ \left( \sigma_{g}^{(t)} \right)^2 = \frac{\sum_{i \in S} \left( Y_i - \mu_{g}^{(t)} \right)^2 P \left( \lambda_i = g | Y_i, \lambda_{N(i)}, \theta^{(t-1)}, T^{(t-1)} \right)}{\sum_{i \in S} P \left( \lambda_i = g | Y_i, \lambda_{N(i)}, \theta^{(t-1)}, T^{(t-1)} \right)} \]

\[ T^{(t)} = \arg \max_{\{T\}} \sum_{i \in S} \sum_{g=0}^{G} \log P \left( \lambda_i = g | \lambda_{N(i)}, T \right) P \left( \lambda_i = g | Y_i, \lambda_{N(i)}, \theta^{(t-1)}, T^{(t-1)} \right) \]
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- approximate in $P(\lambda|Y) \propto P(\lambda)f(Y|\lambda)$ the prior probability using mean field theory \(^{\text{Celeux et al., 2003}}\)

\[ P(\lambda) \approx \prod_{i \in S} P \left( \lambda_i | E[\lambda_{N(i)}] \right) \]
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  \[P(\lambda) \approx \prod_{i \in S} P(\lambda_i | E[\lambda_{N(i)}])\]
- EM iterates between updating \(E[\lambda]\) and parameters

\(^a\) Celeux et al., 2003
Results
Results - MFA

- fit the Gaussian hidden FRAME model to TCA image
- use $\mathcal{G} = \{0, 1\}$ (black and white rings)
- MFA of cementum band at last iteration ($\hat{T} = 14$)
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Results - tooth ring count

- expected # rings: 33.61
- recognized: 35
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- additional TCA images

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<td>1692</td>
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<td>34.39</td>
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- bad detection of rings
Conclusions
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- aim: estimate the number of tooth rings in TCA images
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- results
  - fitted model with long-range dependencies to large images
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\textsuperscript{a}[Celeux et al., 2003]
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  - members of MPI DR tooth lab: A. Fabig and U. Cleven
  - supervisor at the MPI DR: F. Lagona

\(^{a}\)[Celeux et al., 2003]
Thank you!
Bibliography


